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INVESTIGATION OF THE BOUNDARY CONDITION
AT A PERFORATED WALL

by

Paul F. Maeder

Technical Report WT-9

DIVISION OF ENGINEERING
BROWN UNIVERSITY
PROVIDENCE, RHODE ISLAND

May 1953

Sponsored by:

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United States Navy

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PREFACE

The work in this report was done under the sponsorship of the Office of Naval Research and the Office of Scientific Research of the Air Research and Development Command, under Contract N7onr-35805.

Acknowledgement is due Mr. James B. Carroll who performed the tests and prepared the final report for publication.

Brown University
Engineering Research Laboratory
Providence, Rhode Island
May 1953

ABSTRACT

The boundary condition to be applied to the flow along a perforated wall is investigated theoretically using potential flow theory. The theory is confirmed by subsequent experiments with a low speed tunnel. The boundary condition thus found is linear and not quadratic as was previously supposed.

TABLE OF CONTENTS

SYMBOLS

α	Flow coefficient
β	Prandtl-Glauert factor
c	Velocity
Γ	Circulation
M	Mach number
p	Pressure in test section
p_{∞}	Pressure in plenum chamber
ϕ	Complex potential
Q	Source strength
ρ	Density
σ	Ratio of open area to total area
U	Undisturbed flow velocity
u,v,w	Velocity components due to opening
N_{Re}	Reynolds number
m	Constant of proportionality

FIGURES

I. INTRODUCTION

In the theoretical treatment of the potential flow inside a wind tunnel employing perforated walls to reduce wall interference in the lower transonic regime, there is some question as to what boundary conditions should be imposed onto the flow at the wall.

The first theoretical work (Ref. 1) assumed walls of a porous rather than a perforated material. Consequently, the flow through the wall was taken as viscous and a linear pressure velocity relationship was assumed.

$$\bar{v} = -K\Delta p \tag{1}$$

Extensive investigations were carried out for this boundary condition.

Subsequently, in actual experimental investigations, a perforated wall was substituted for the porous one. For this wall the hole size is no longer small compared to the wall thickness, and the treatment of the individual holes as orifices is at once suggested. This seems to indicate a quadratic pressure-velocity relationship:

$$\bar{v} = \alpha \sqrt{\frac{2}{\rho}\Delta p} \tag{2}$$

$\alpha \approx .62$ for thin plate orifice.

However, such a treatment assumes that the pressure difference Δp at the hole cross section is created by the velocity components u, v, and w which are caused by the hole and are of equal order of magnitude, i.e., u and w at a given point are proportional to v. The exact theoretical treatment thus can yield an orifice coefficient α as constant of proportionality of v. While this is correct if there is no appreciable flow in the x direction, it is certainly no longer true in a wind tunnel where there is a main flow component U.

If the latter is included in the considerations, the pressure difference between hole cross section and tunnel main flow measured at a distance from the wall which is large compared to the hole diameter becomes:

$$\Delta p = \frac{\rho}{2} c^2 - \frac{\rho}{2} U^2 \tag{3}$$

where c is the velocity at the hole. If the velocity components of the perturbation velocity $\vec{c} - \vec{U}$ are denoted by u, v, w, one obtains

$$\Delta p = \frac{\rho}{2} \left\{ (U+u)^2 + v^2 + w^2 + U^2 \right\}$$

$$= \rho U u + \frac{\rho}{2} (u^2 + v^2 + w^2)$$

Since u, v, w are of equal order of magnitude, one sees that if U is much larger than v and w the pressure in the hole is determined by the first term in the above equation.

$$\frac{\Delta p}{\frac{\rho}{2} U^2} = 2 \frac{u}{U} \qquad 2U > u \sim v \sim w \tag{4}$$

Thus, since u is proportional to v and thus also to $\bar{v}$, the mean value over the hole, a linear pressure-velocity relationship is obtained for the perforated wall.

In the following sections, the constant of proportionality between u and $\bar{v}$ will be calculated for the case of transverse slots in the tunnel wall, assuming a constant Δp over the slot cross section. This latter assumption is the same as the one made for the calculation of the flow through an orifice, the results of which have been verified experimentally.

It will also be shown that this constant depends on β for linearized subsonic compressible flows in such a way as to cancel the β obtained in the required boundary condition for interference free walls. Thus one and the same perforated wall should yield interference free results within the whole range of linearized subsonic theory.

The theoretical results thus obtained will be compared with experiments performed in a low speed tunnel on various boundaries including perforated walls.

II. THEORETICAL INVESTIGATION

A. The Two Dimensional Transverse Slot of Constant Pressure

Consider the problem as shown in Figure 1. At $y = 0$ there is a slot in a solid wall extending from $x = -1$ to $x = 1$. The pressure over the slot width shall be constant and equal to $p_{\infty} = p + \Delta p$. Thus, by virtue of the linearized pressure relationship, (equation 4) there will be a constant velocity u_o extending from $x = -1$ to $x = 1$.

The problem is therefore the one of finding a potential $\Phi(z)$ yielding a constant horizontal velocity in the slot, and no vertical velocity along the remainder of the x axis.

$$\text{Thus: at } y = 0 \qquad -1 < x < 1 \qquad u = u_o$$

$$\left.\begin{matrix} x < -1 \\ x > 1 \end{matrix}\right\} \qquad v = 0$$

$$\text{at } z \rightarrow \infty: \qquad \Phi'(z) \rightarrow 0$$

If this potential is multiplied by i the problem is transformed to:

$$\Phi(z) = i\,\Phi(z)$$
$$\Phi_2'(z) = i\,\Phi'(z) = u_2 - i\,v_2 = v + i\,u \qquad (5)$$
$$u_2 = v \qquad\qquad v_2 = -u$$

The boundary conditions then become:

at $y = 0$	$-1 < x < 1$	$v_2 = -u_0 =$ constant
	$x < -1$	$u_2 = 0$
	$x > 1$	

This is at once recognized as the perturbation flow field generated by a flat plate moving downward with a constant velocity u_0 (Figure 2).

The potential of the flat plate is obtained by applying the Joukowski transformation to the flow about a cylinder. In doing this, finite velocities at the points $x = \pm 1$ become infinite. In examining the original problem one finds that an infinite velocity at $x = -1$ is an impossibility, since the wall possesses at this point what one might call a trailing edge. In this point the Joukowski condition has to be applied. This at once limits the number of solutions of the flow about the cylinder to the one which possesses a stagnation point at $x = -1$.

The flow about a cylinder blown at in the vertical direction obeys the following potential in the ζ plane. (Figure 3).

$$\Phi_2(\zeta) = -\frac{iM}{2\pi\zeta} + \frac{i\Gamma}{2\pi}\ln\zeta \qquad (6)$$

This flow shall have a stagnation point at $\zeta = -1$.

at $\zeta = -1$ $\quad \phi_2'(\zeta) = 0$ $\quad$ and

$$\frac{iM}{2\pi} - \frac{i\Gamma}{2\pi} = 0$$

Since the potential of Γ, a circulation, if multiplied by i yields the potential of a source, Γ can be replaced by Q.

$$Q = \Gamma = M$$

The potential becomes:

$$\Phi(\zeta) = -i\,\Phi_2(\zeta) = \frac{Q}{2\pi}\left(\ln\zeta - \frac{1}{\zeta}\right) \tag{7}$$

The Joukowski transformation relates this to the z plane by:

$$z = \frac{1}{2}\left(\zeta + \frac{1}{\zeta}\right) \qquad \zeta = z \pm \sqrt{z^2 - 1} \tag{8}$$

If this is applied to the potential $\Phi(\zeta)$ one obtains for the velocities:

$$u - iv = \Phi'(z) = \Phi'(\zeta)\frac{d\zeta}{dz} = \frac{Q}{2\pi}\left(\frac{1}{\zeta^2} + \frac{1}{\zeta}\right)\frac{2\zeta^2}{\zeta^2 - 1}$$

$$= \frac{Q}{\pi}\;\frac{1}{\zeta - 1}$$

$$= \frac{Q}{2\pi}\left\{\pm\sqrt{\frac{z+1}{z-1}} - 1\right\} \tag{9}$$

where the sign is to be determined from case to case. Along the wall, at y = 0

$$|x| > 1 \qquad u = \frac{Q}{2\pi}\left\{\sqrt{\frac{x+1}{x-1}} - 1\right\} \tag{10a}$$

$$v = 0 \tag{10b}$$

$$|x| < 1 \qquad u = -\frac{Q}{2\pi} \tag{11a}$$

$$v = \mp \frac{Q}{2\pi}\sqrt{\frac{1+x}{1-x}} \tag{11b}$$

+ for upper half plane.

- for lower half plane.

The mean vertical velocity through the slot can be obtained by integration.

$$\bar{v} = \frac{1}{2}\int_{-1}^{+1} v \cdot dx = -\frac{Q}{4} \tag{12}$$

The velocity distribution about the slot, as calculated from equations (10a, 10b) (11a, 11b) is shown in Figure 4.

If the Goethert rule is applied to the above results one obtains,

$$u = \frac{u_i}{\beta} \qquad v = \frac{v_i}{\beta} \qquad Q = \frac{Q_i}{\beta}$$

then at

$$|x| < 1 \qquad u = -\frac{Q}{2\pi\beta}$$

$$v = -\frac{Q}{2\pi}\sqrt{\frac{1+x}{1-x}}$$

$$\bar{v} = -\frac{Q}{4} \qquad \frac{\pi}{2} \; \beta u \tag{13}$$

B. Infinite Number of Transverse Slots

If the wall is considered as containing an infinite number of transverse slots (Figure 5), uniformly spaced along the x axis, the method described in Reference 2, pages 77 to 93, can be applied to obtain the velocities due to these slots.

It can be said that the flow must have suction points at $z = n\ell_2 \pm \ell_1$. The stagnation points must be located in such a way that the velocities at the trailing edges of the solid portions will remain finite. For this condition to be realized, terms like $z - (n\ell_2 - n\ell_1)$ must appear in the numerator. If the sine is used to replace the infinite products, the conjugate velocity c^* will be given by

$$c^* = u - iv = a\sqrt{\frac{\sin\frac{\pi}{\ell_2}(z+\ell_1)}{\sin\frac{\pi}{\ell_2}(z-\ell_1)}} + b \tag{14}$$

where a and b are complex constants to be determined from the boundary conditions:

at $y = 0$

in the interval $0 < x < \ell_1$: $c^* = u_0 - iv$ $u_0 = \text{const.}$

but: $$\frac{\sin\frac{\pi}{\ell_2}(z+\ell_1)}{\sin\frac{\pi}{\ell_2}(z-\ell_1)} < 0$$

Thus the square root becomes imaginary and, since the real part of c^* must be a constant, a must be real.

$$\text{Im } a = 0 \qquad \text{Re } b = u_0$$

At $y = 0$:

in the interval $\ell_1 < x < \ell_2 - \ell_1$: $c^* = u$

But since the square root is real in this interval and the velocity shall not contain any imaginary part, we conclude that

$$\text{Im } b = 0$$

Thus a and b become:

$$a = \text{Re } a$$

$$b = u_0$$

The value of a can be related to the mean vertical velocity by evaluating the velocities at a large distance from the wall.

as $y \to \infty$: $\qquad c^* = -iv'$

However: $$\lim_{y \to \infty} \sqrt{\frac{\sin \frac{\pi}{\ell_2}(z+\ell_1)}{\sin \frac{\pi}{\ell_2}(z-\ell_1)}} = \cos \pi \frac{\ell_1}{\ell_2} + i \sin \pi \frac{\ell_1}{\ell_2} \tag{15}$$

and: $$v' = -a \sin \pi \frac{\ell_1}{\ell_2} \tag{16}$$

$$u_0 = -a \cos \pi \frac{\ell_1}{\ell_2} \tag{17}$$

The velocity relationship now becomes:

$$v' = u_0 \tan \pi \frac{\ell_1}{\ell_2} = u_0 \tan \frac{\pi \sigma}{2} \tag{18}$$

Again, extending the analysis by means of the Prandtl-Glauert rule to the linearized subsonic case, one obtains

$$\frac{v'}{U} = \beta \frac{u_0}{U_\infty} \tan \frac{\pi \sigma}{2} = \frac{\beta \Delta p}{2q} \tan \frac{\pi \sigma}{2} \tag{19}$$

Reference 1 gives the boundary condition required to have no interference as:

$$\frac{v'}{u} = -R = -\beta \cot \frac{\pi}{2\sqrt{3}} = .781\beta \tag{20}$$

From equation 20, the ratio of open to total boundary σ, can then be evaluated independently of Mach number. The numerical value for an infinite number of transverse slots is $\sigma = 42.2\%$.

III. EXPERIMENTAL INVESTIGATION

A. Apparatus

A schematic diagram of the testing equipment is shown in Figure 6. A vane blower was used to supply air at a dynamic pressure of approximately 2.3 cm. H_2O, to an 8" x 4" test section. The two side walls of the test section were solid while the lower side was left open to the atmosphere to insure atmospheric pressure in the free stream. The upper side of the test section was arranged to accommodate different types of walls. The upper walls were made of 1/4" aluminum and were of the following types.

(a) Wall containing one 1" x 4" transverse slot, beveled 45°, sharp edge to free stream.

(b) Wall containing circular hole of 4 in^2 area.

(c) Wall containing 4" x 5" opening into which various samples of perforated sheets could be fitted.

Over the opening in the upper wall a plenum chamber was constructed. Auxiliary suction equipment was available so that both positive and negative pressure differences could be obtained in the plenum chamber. The quantity of air flowing through the wall was measured by a precision metering nozzle of 1" diameter.

The plenum chamber was equipped with screens and guide walls so that the flow through the upper wall would be essentially two dimensional. Pressure taps were provided so that the tank pressure could be measured. Figure 7. shows an isometric view of the plenum chamber.

The perforated metal inserts were made of brass, the perforations being of two patterns, square and hexagonal. Two tests were made for each hexagonal pattern, the second test being made on a perforated sheet whose hexagonal pattern was rotated 90° from the axis of the original sheet.

All pressure measurements were made on a "Betz Type" water manometer, which had an absolute error of .002 cm H_2O.

B. Data Reduction

The following three quantities were measured.

1. Dynamic pressure q
2. Plenum chamber pressure difference $p_\infty - p = \Delta p$
3. Pressure difference across orifice Δp_{or}

Knowing these three quantities the following dimensionless quantity could be evaluated.

$$\frac{\bar{v}}{U} = \frac{\alpha A_N}{\sigma A_s} \sqrt{\frac{\frac{2}{\rho}\Delta p_{or}}{\frac{2}{\rho} q}}$$

$$= \frac{\alpha \pi}{\sigma A_s 4} \sqrt{\frac{\Delta p_{or}}{q}}$$

A_N = Nozzle area

A_S = Area of wall opening

α = Nozzle coefficient

Using these equations it was possible to plot the wall characteristic curves $\frac{\bar{v}}{U}$ vs. $\frac{\Delta p}{q}$.

C. Description of Tests

The first test was made to confirm the assumption that the flow through the slot is essentially irrotational as assumed by theory. Static pressure taps were placed up and downstream of the slot along the x axis. From the pressure distributions thus obtained, it was possible to calculate the velocity distribution and compare it to the theoretical distribution given by equations 10a, 11a.

Once the accuracy of the theory was determined, the pressure velocity characteristics of this slot, the single hole and different samples of perforated inserts in the upper wall were determined by measuring Δp, Δp_{or} and q.

As an additional experiment the influence of the initial boundary layer was determined by conducting tests on perforated metal inserts of varying lengths. The lengths tested ranged from 5 inches to 1 inch.

IV. RESULTS OF TESTS

A. Confirmation of Theory for Transverse Slot

Figure 8. shows the curves of the theoretical and experimental velocity distributions in the region of the slot. The two curves correspond to a sufficient degree to admit the assumption of potential flow through the slot.

B. Wall Characteristics

1. Rectangular 4" x 1" Slot

Figure 9. shows a comparison between the theoretical and experimental characteristics of the slot. The curve labeled $\left(\frac{\Delta p}{q}\right)_{\text{th linear}}$ is a plot of equ. 4, while the curve labeled $\left(\frac{\Delta p}{q}\right)_{\text{th exact}}$ is a plot of the same equation taking into account quadratic terms in Bernoulli's equation. Since in the actual use of perforated walls, the perturbation term $\frac{\bar{v}}{U}$ is smaller than 10%, the linear law gives a sufficiently accurate approximation.

2. Circular Hole

The results of tests made with a circular hole in the upper wall indicate that it also has a linear characteristic. Because of the influence of the side walls, these results correspond to a row of circular holes four inches apart.

3. Perforated Walls

Sample wall characteristic curves are shown in Figures: 10, 11, 12, 13, 14. Again it appears that the relation is a linear one within the limits of the proposed theory.

It is of interest to note the displacement of the experimental curves to the right of the origin. This can be explained in the following way. When $\Delta p = 0$, a mixing process will exist similar to that occurring in a free jet. Due to this mixing process a vertical velocity will be induced. As Δp goes to negative values, the outflow through the wall will become more and more restricted, until at the point where the characteristic curve crosses the $\frac{\Delta p}{q}$ axis, the adverse pressure difference will tend to cancel any outflow.

4. Initial Boundary Layer Influence

Results of boundary layer influence tests are presented in Figure 15. It is noted that the linearity is not affected by the changes in the ratio of boundary layer thickness to length of insert. The constant of proportionality is affected only to a slight degree.

V. CONCLUSIONS

It is at once evident from the results that the boundary condition for the perforated wall is a linear one, and not quadratic as was previously supposed.

The Reynolds number which varied over a large range of values does not affect the linearity. From Figure 16. it is seen that the orientation of the hexagonal pattern does have some influence on the constant of proportionality m.

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1. Theodore R. Goodman — The Porous Wall Wind Tunnel. Part II. Interference Effect on a Cylindrical Body in a Two-Dimensional Tunnel at Subsonic Speed. Cornell Aeronautical Laboratory, Inc. Report No. AD-594-A-3, November 1950.

2. R. Grammel — Die Hydrodynamischen Grundlagen des Fluges Friedrich Vieweg Braunschweig 1917.

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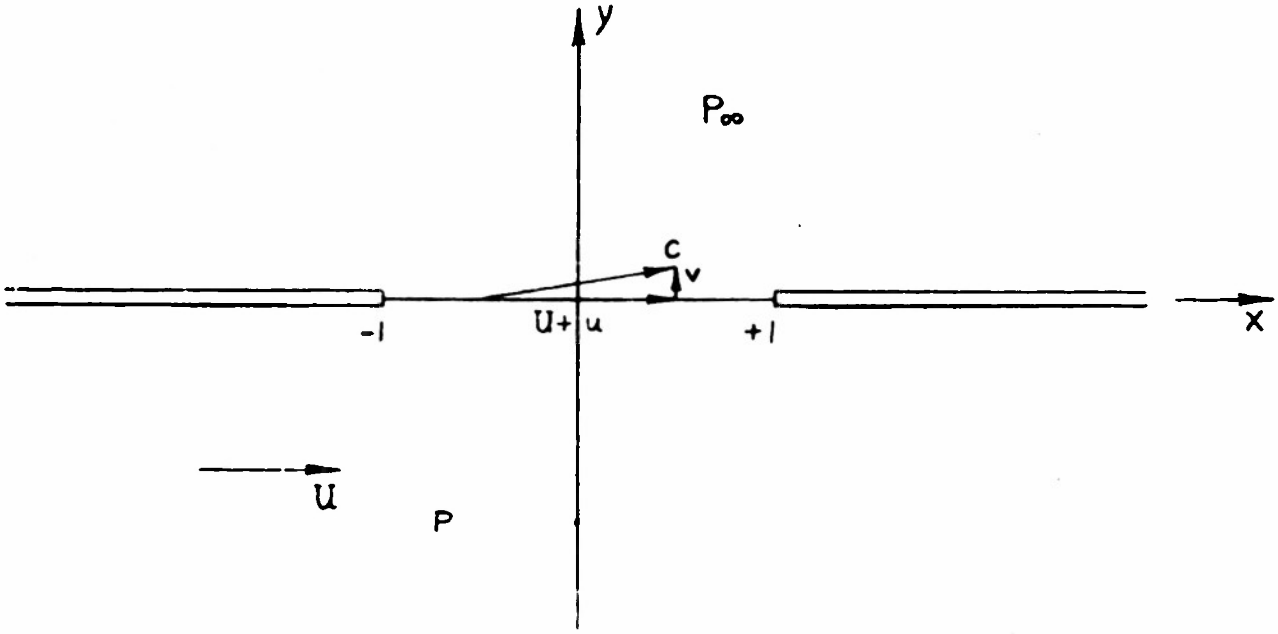
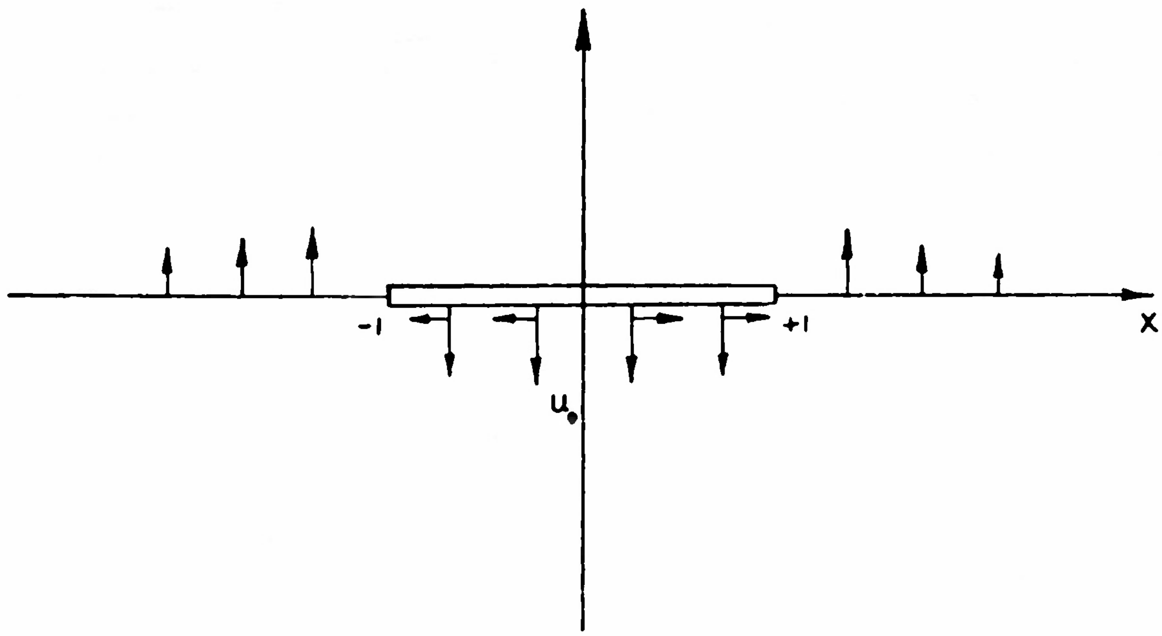


VELOCITIES THROUGH A TWO DIMENSIONAL SLOT

FIG 1



FLOW ABOUT A FLAT PLATE MOVING DOWNWARDS WITH CONSTANT VELOCITY

FIG. 2

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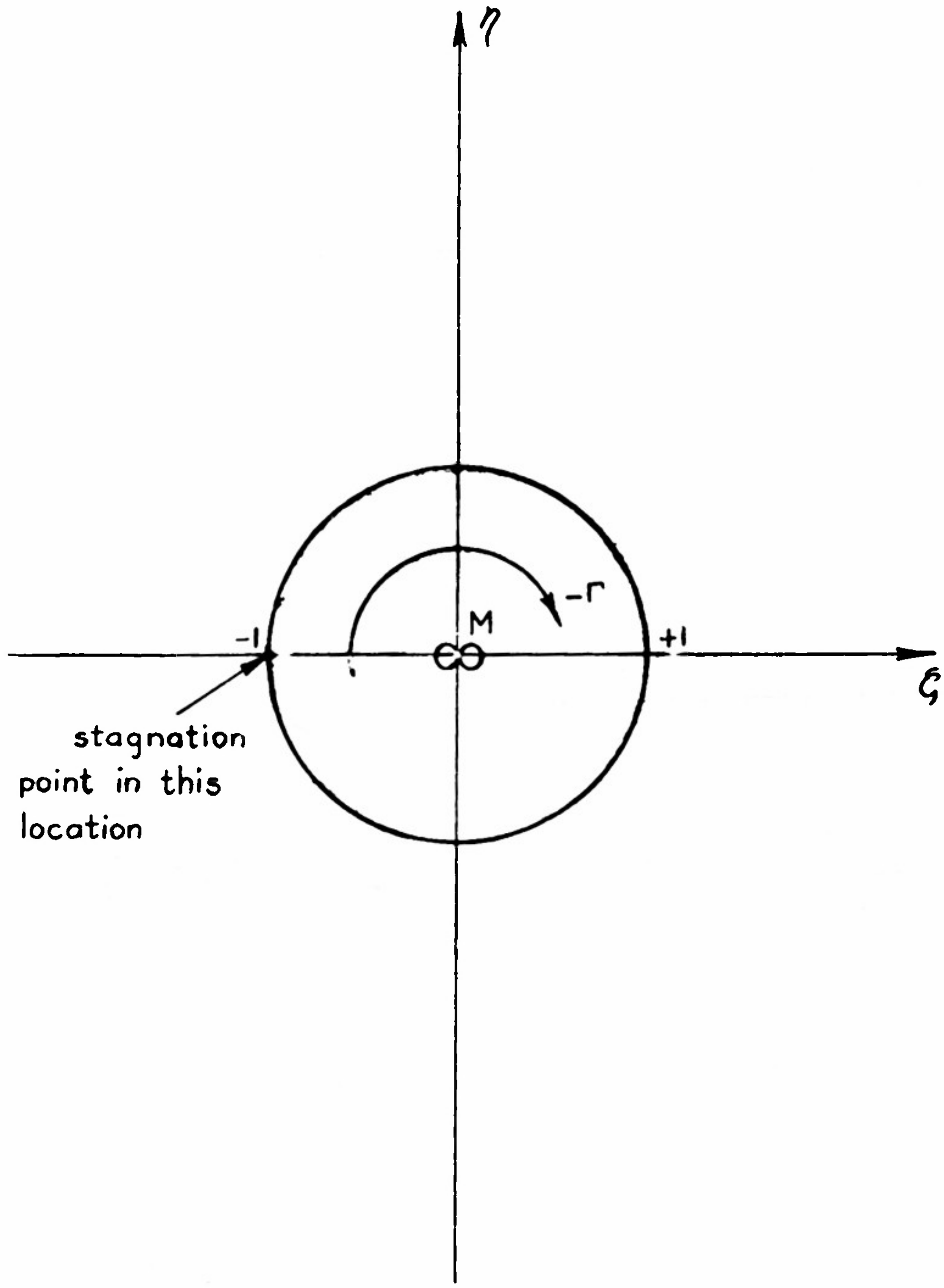

TRANSFORMATION OF SLOT TO CYLINDER

FIG. 3

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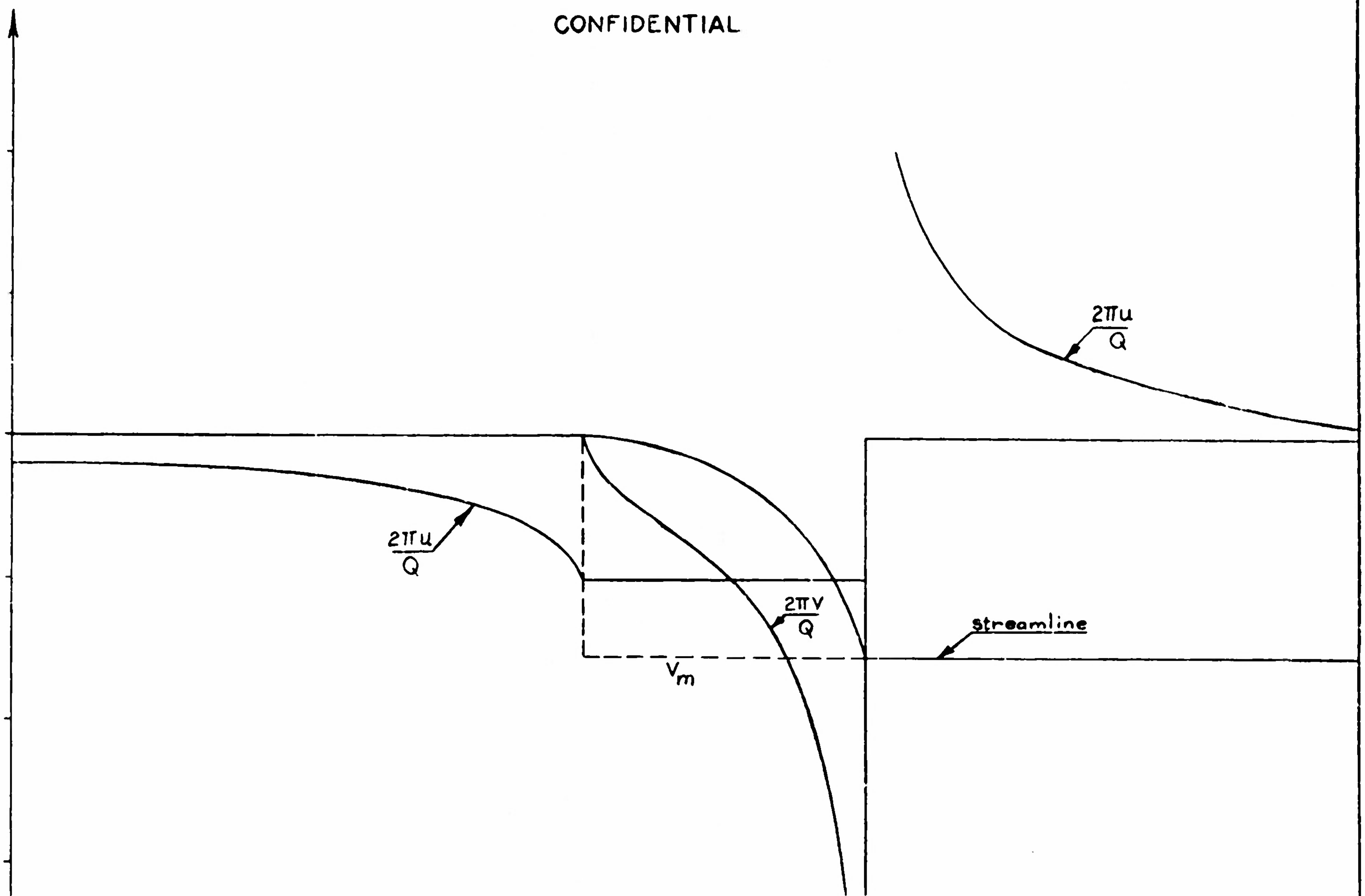

THEORETICAL VELOCITY DISTRIBUTION NEAR SLOT

FIG. 4

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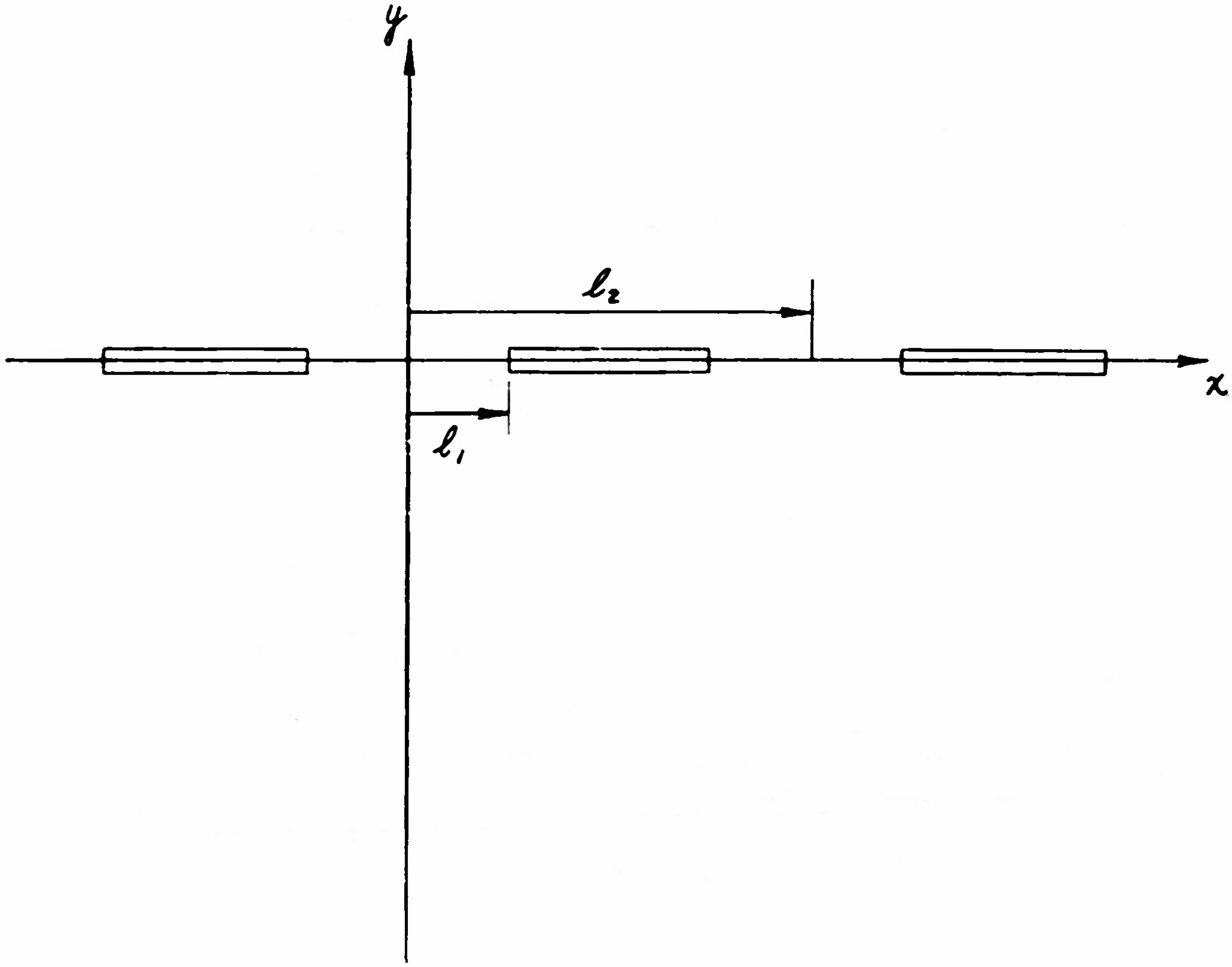

WALL CONTAINING INFINITE NUMBER OF SLOTS

FIG. 5

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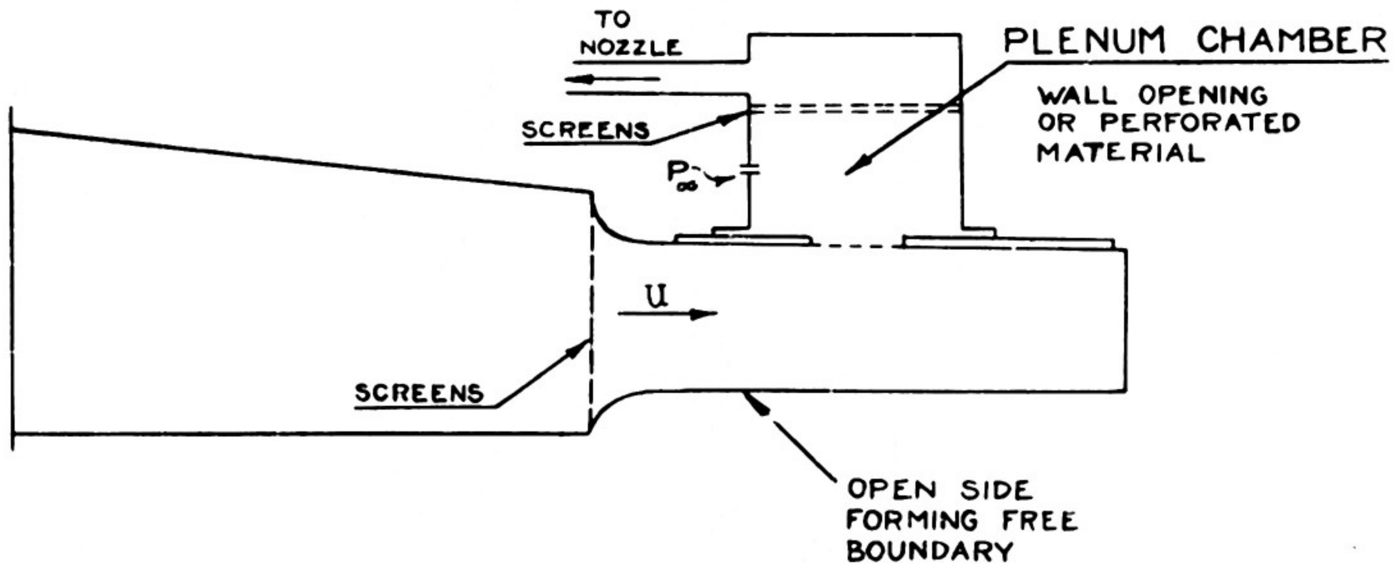

SCHEMATIC DIAGRAM OF TESTING ARRANGEMENT

FIG 6

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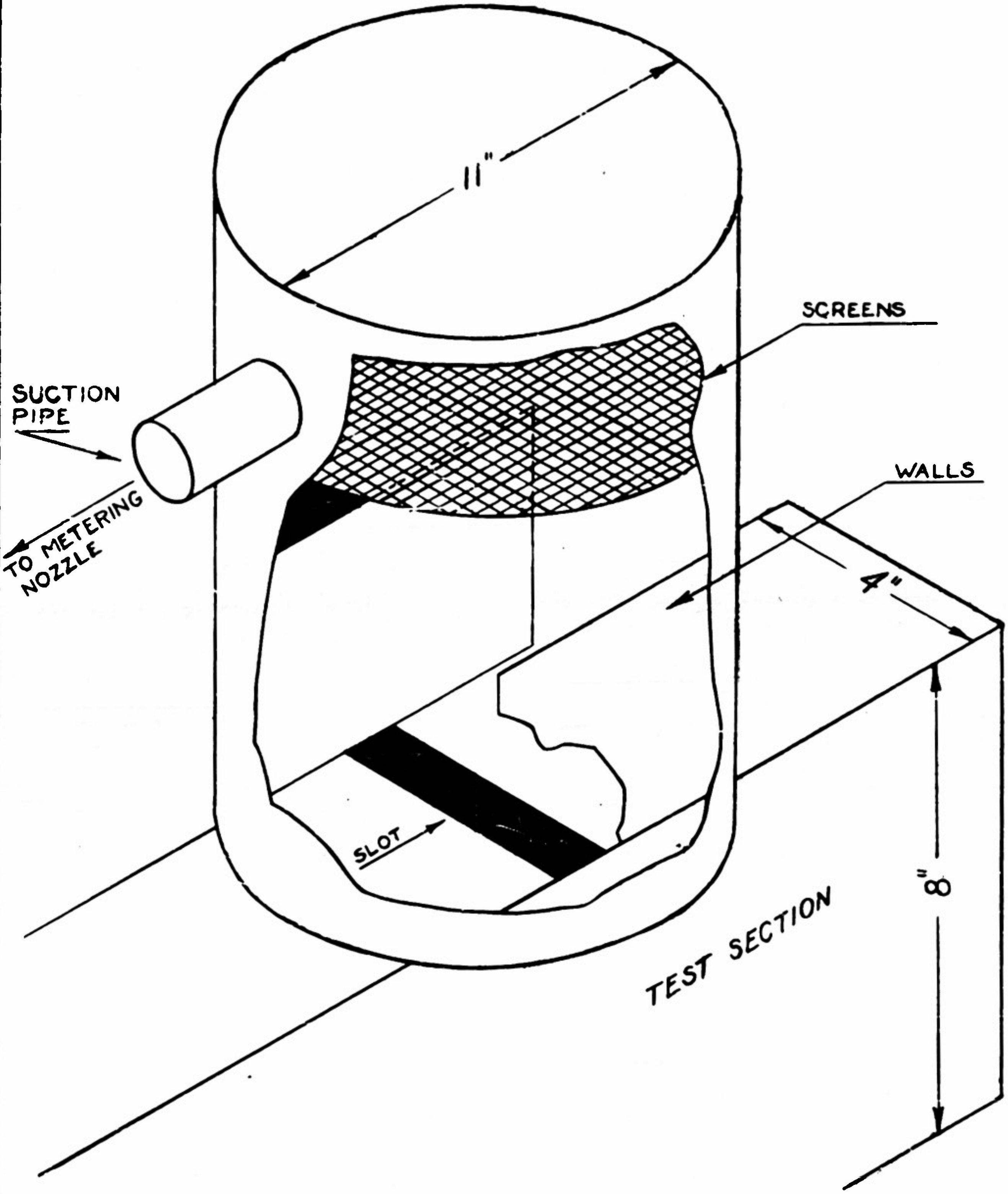

ISOMETRIC VIEW OF PLENUM CHAMBER

FIG. 7

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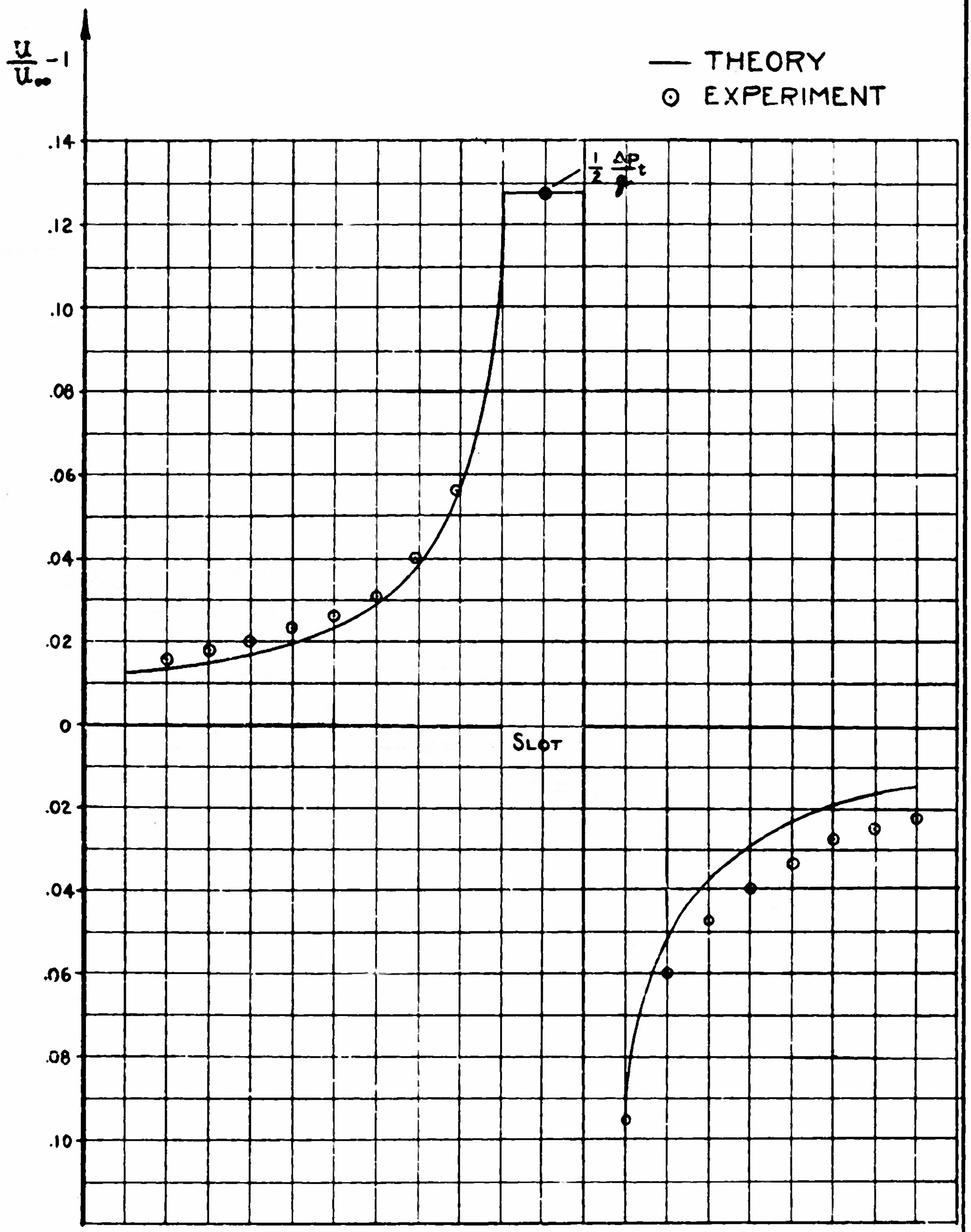

VELOCITY DISTRIBUTION
ALONG WALL

FIG. 8

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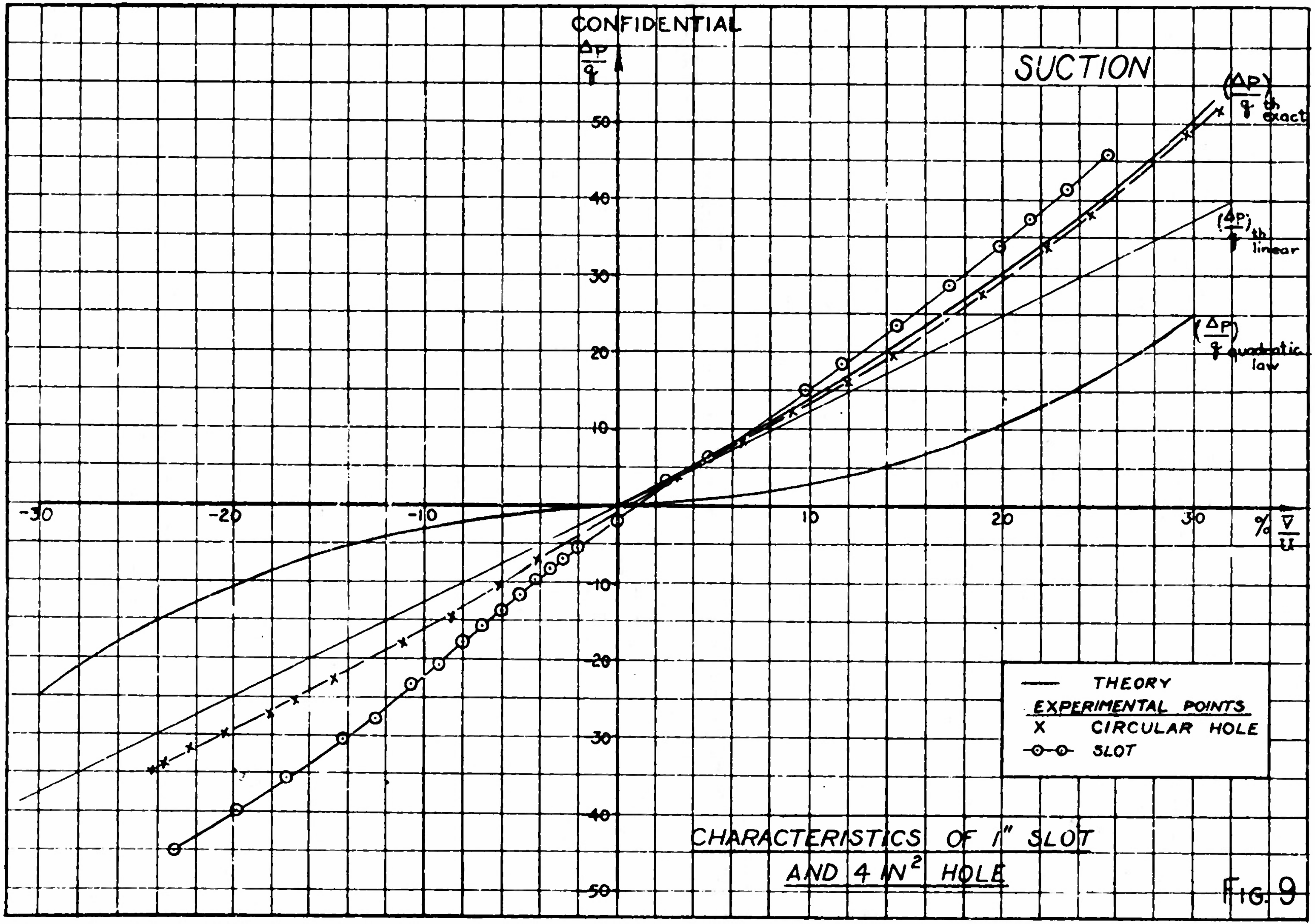
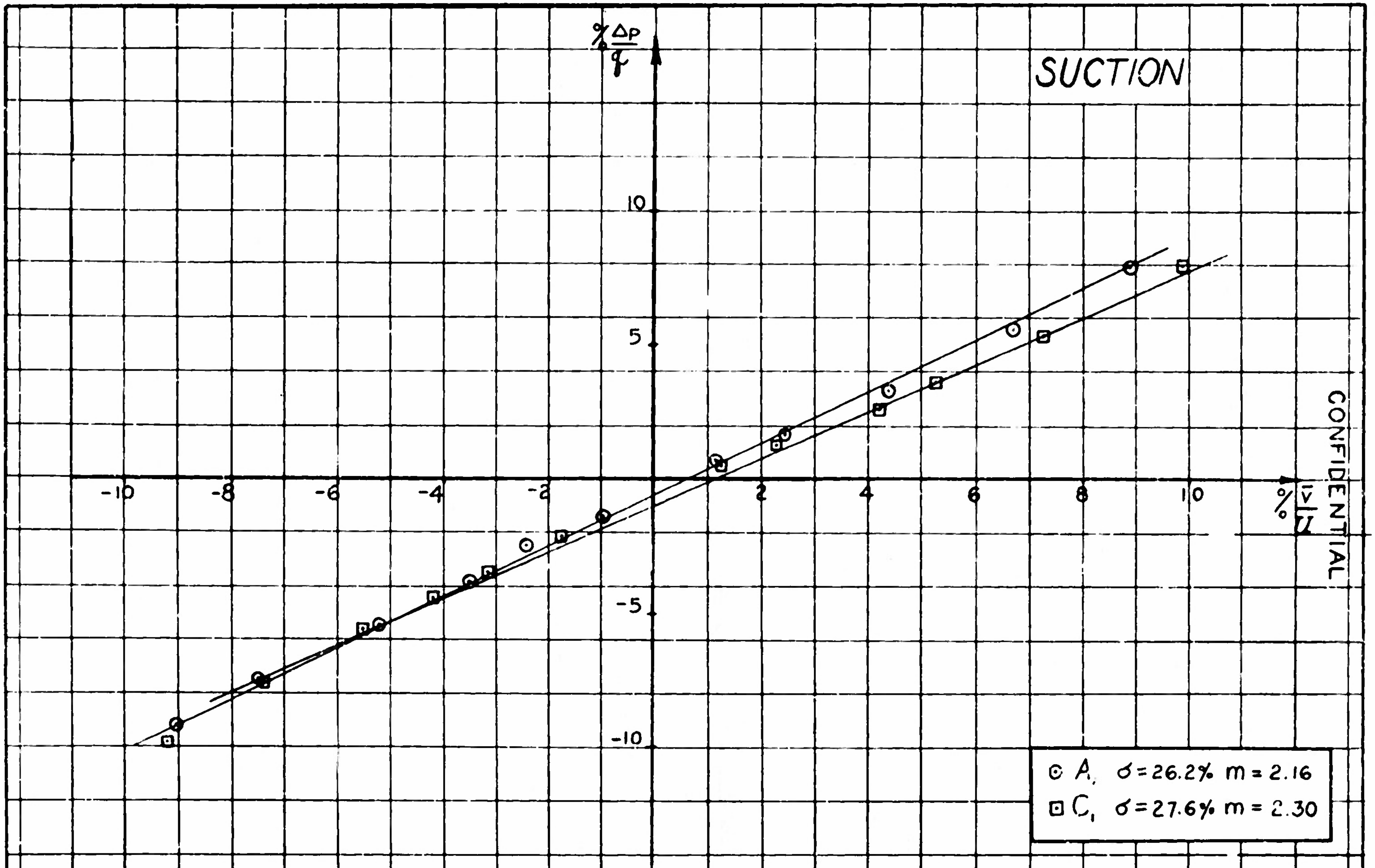


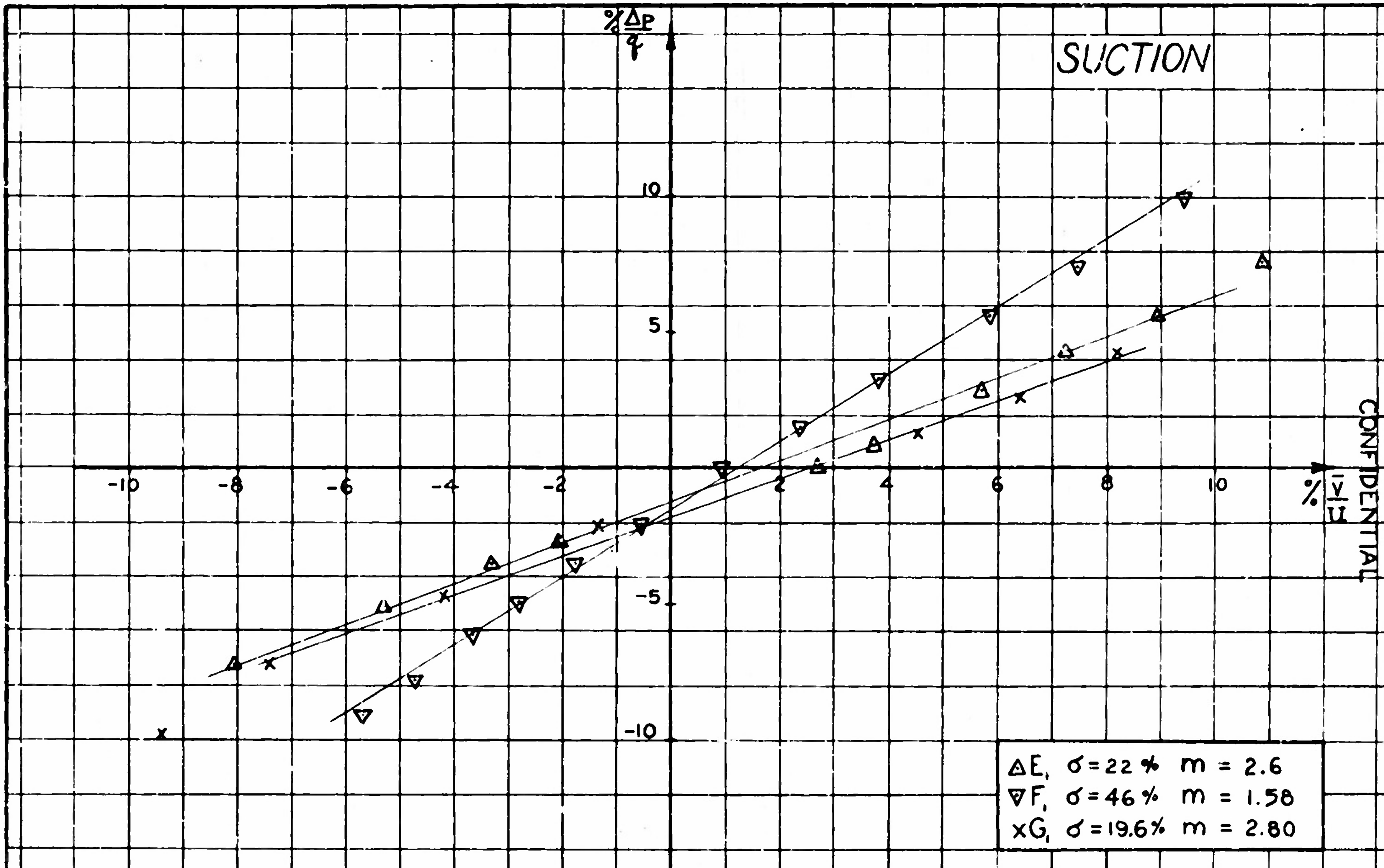
CHARACTERISTICS OF 1" SLOT
AND 4 IN^2 HOLE

FIG. 9



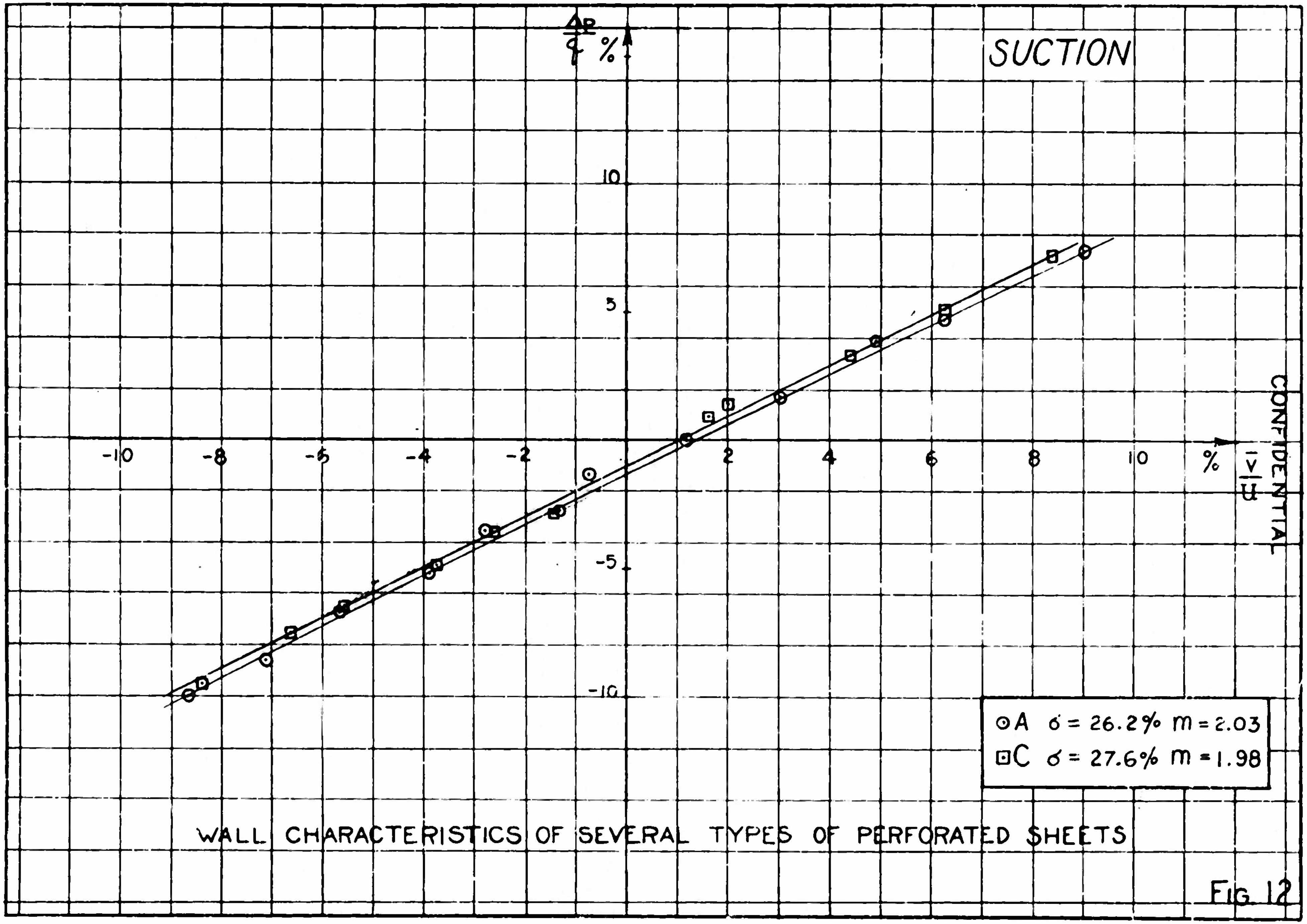
WALL CHARACTERISTICS OF SEVERAL TYPES OF PERFORATED SHEETS

FIG. 10



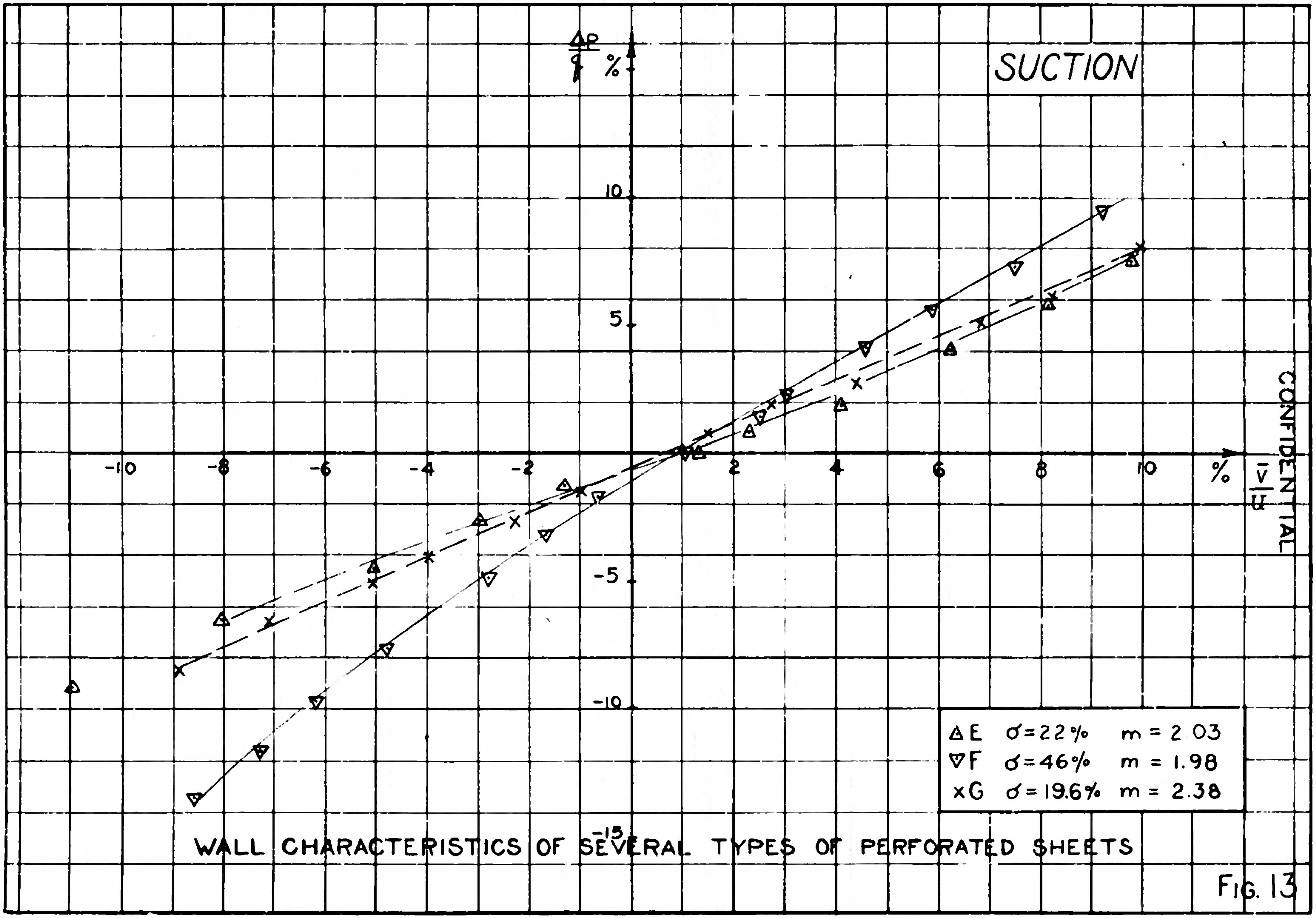
WALL CHARACTERISTICS OF SEVERAL TYPES OF PERFORATED SHEETS

FIG. 11



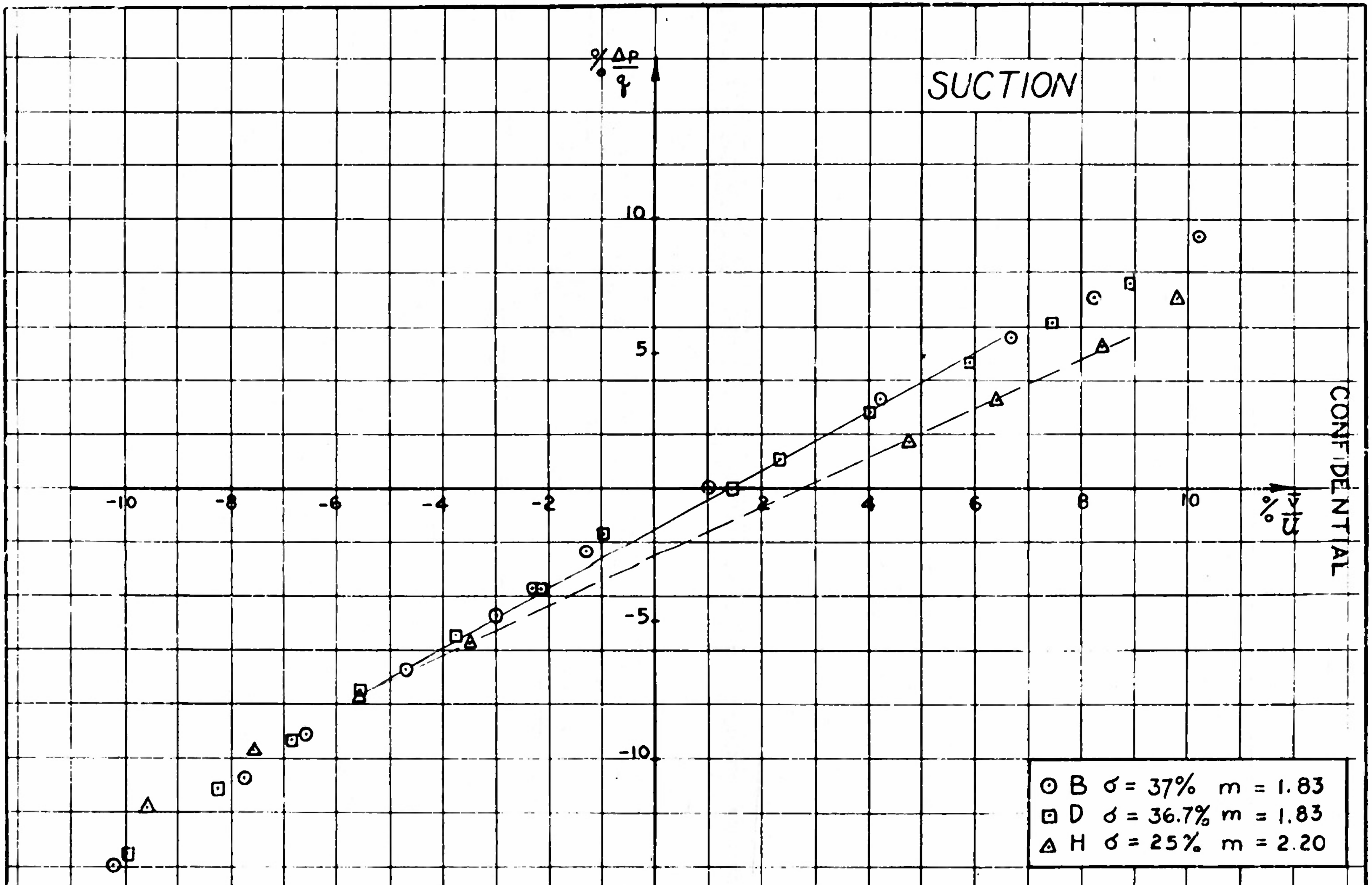
WALL CHARACTERISTICS OF SEVERAL TYPES OF PERFORATED SHEETS

FIG. 12



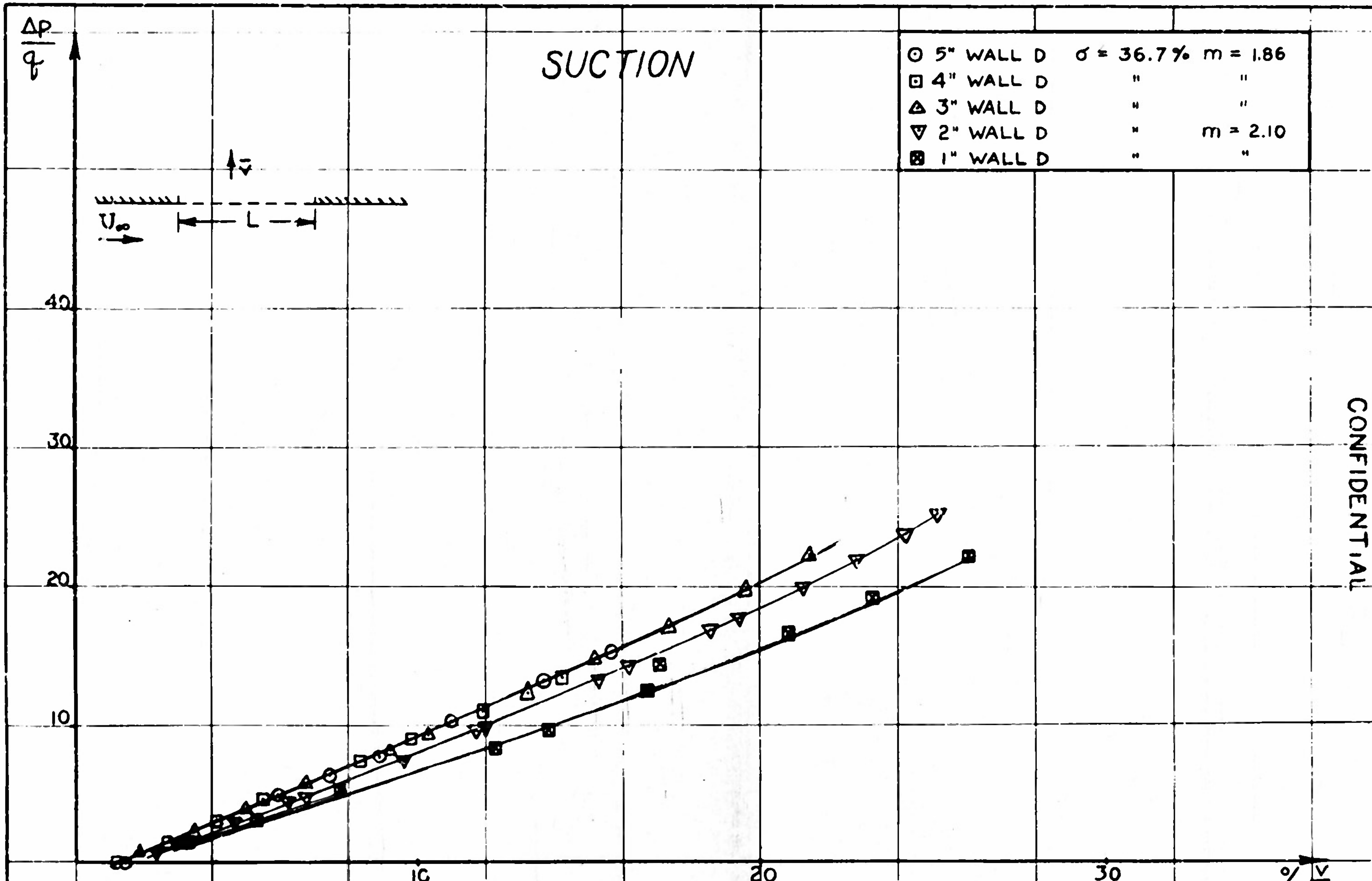
WALL CHARACTERISTICS OF SEVERAL TYPES OF PERFORATED SHEETS

FIG. 13



WALL CHARACTERISTICS OF SEVERAL TYPES OF PERFORATED SHEETS

FIG. 14



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CHARACTERISTICS OF PERFORATED SHEET
VARYING THE LENGTH OF SHEET

FIG. 15

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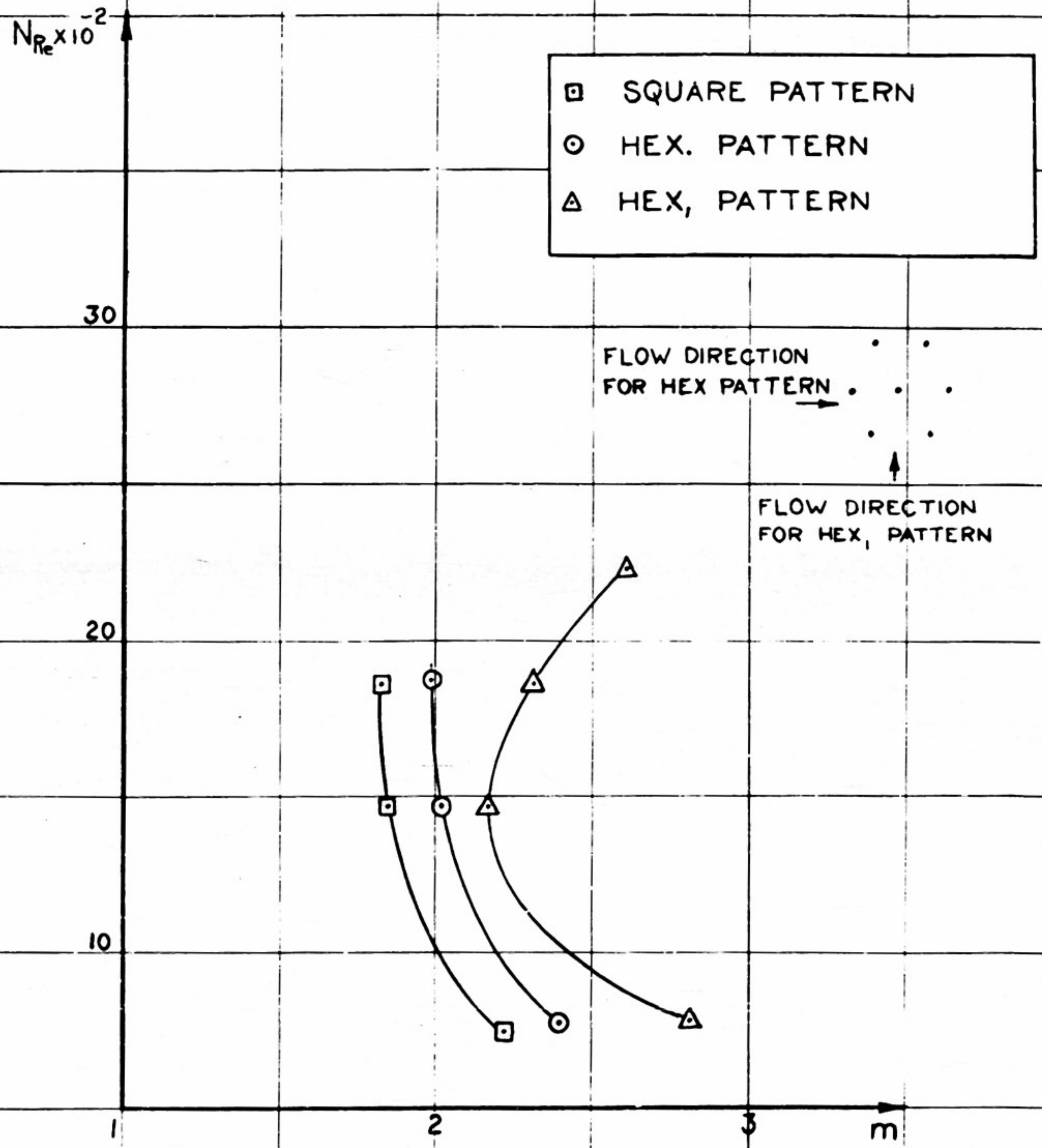

VARIATION OF m WITH REYNOLDS NUMBER

FIG. 16

	Diameter of Holes	Holes/in^2	Thickness	Arrange-ment	σ %	N_{Re}	m
4"x 1" Slot	4" x 1"		.25			33,000	1.72
Cir. Hole	2.26"		.25			132,000	1.365
Wall A	.045	165	.0325	Hex.	26	1,490	2.03
Wall A_1	.045	165	.0325	$Hex._1$	26	1,490	2.16
B	.045	225	.0320	Square	37	1,490	1.83
C	.057	108	.0325	Hex.	27.6	1,850	1.98
C_1	.057	108	.0325	$Hex._1$	27.6		2.30
D	.057	144		Square	36.7	1,850	1.83
E	.068	56	.022	Hex.	22	2,300	2.76
E_1	.068	56	.022	$Hex._1$	22		2.6
F	.077	92	.031	Hex.	46	2,300	
F_1	.077	92	.031	$Hex._1$	46		7.58
G	.024	432	.022	Hex.	19.6	800	2.38
G_1	.024	432	.022	$Hex._1$	19.6		2.80
H	.023		.0183	Square	25		2.20

TABLE I